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# CALIBRATION OF GIMBALED PLATFORMS: THE SOLAR DYNAMICS OBSERVATORY HIGH GAIN ANTENNAS

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## Abstract

Simple parameterization of gimbaled platform pointing produces a complete set of 13 calibration parameters—9 misalignment angles, 2 scale factors and 2 biases. By modifying the parameter representation, redundancy can be eliminated and a minimum set of 9 independent parameters defined. These consist of 5 misalignment angles, 2 scale factors, and 2 biases. Of these, only 4 misalignment angles and 2 biases are significant for the Solar Dynamics Observatory (SDO) High Gain Antennas (HGAs).

An algorithm to determine these parameters after launch has been developed and tested with simulated SDO data. The algorithm consists of a direct minimization of the root-sum-square of the differences between expected power and measured power.

The results show that sufficient parameter accuracy can be attained even when time-dependent thermal distortions are present, if measurements from a pattern of intentional offset pointing positions is included.

## 1. Introduction

SDO[1] will be in an inclined, geosynchronous orbit with continuous contact to dedicated ground antennas at White Sands. In order to maintain continuous contact, the HGAs must accurately point at the ground antennas. After deployment, the antenna mounting positions can differ significantly from design. It is therefore necessary to plan for on-orbit determination of the true, post-deployment mounting angles of the HGAs. The budget for total antenna pointing error after calibration is 0.25 deg.

During launch, the antennas are stowed against the body of the spacecraft and are deployed after release from the launch vehicle. The predicted repeatability of antenna deployment is

about 1 deg. Deployment error alone is expected to exceed the pointing error budget. Clearly, on-orbit gimbal calibration is necessary.

Although gimbaled platforms provide the ability to point an instrument over a wide range of directions, they have pointing errors due to misalignments of several independent segments of the antenna masts. Because the misalignments of segments of the antenna mast are rotated by large-angle gimbal rotations, they cannot generally be combined for simplification. On-orbit alignment calibration presents a problem with a large number of independent parameters and limited information. HGA power, measured at ground antennas, is the only observable that can be used in the loss function needed to calibrate the gimbals.

This paper presents an algorithm for on-orbit gimbaled platform calibration that was developed and tested for the HGAs on SDO. The parameterization is similar to that used for the Solar-Stellar Pointing Platform (SSPP) on the Upper Atmosphere Research Satellite (UARS) [2].

To validate the calibration algorithm, and to investigate the quantity and properties of measurement data needed to perform a calibration, a simple simulator was developed. This simulator provided target antenna power values as a function of SDO position, attitude, and gimbal read-out angles. It was run for numerous cases, representing calibration in different seasons and with different data spans. In addition, the effect of intentionally pointing the HGA to a pattern of offset positions was also investigated. A simple thermal distortion model was included to represent the time-dependent misalignment of the gimbal structure as the antenna to Sun angle changes.

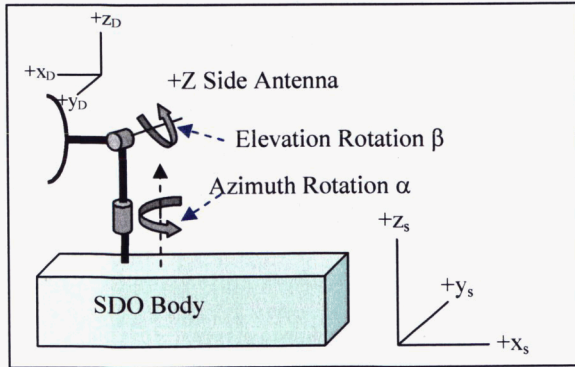
The results showed that the calibration could be both accurate and robust even in the presence of dynamic, thermal distortions of the gimballed platform. Calibration accuracy was greatly improved by intentional off-pointing of the HGAs.

## 2. Parameterization

Gimballed platforms achieve their wide pointing range using gimbals that can rotate over wide ranges. The SDO HGA mast includes two gimbals. Figure 1 shows the orientation of one of the HGAs on SDO. A second HGA is on the  $-z$ -side of the spacecraft.

The azimuth gimbal is designed to rotate about the spacecraft  $z$ -axis and can move in a complete circle. Its rotation angle is designated  $\alpha$ . The elevation gimbal is designed to rotate about an axis perpendicular to that of the azimuth gimbal and has a range of  $\pm 69$  deg. Its rotation angle is designated  $\beta$ .

The dish frame, designated with a subscript “D”, is indicated in Fig. 1. It is attached to the antenna dish and rotates with it. With both gimbals in their nominal zero position, the antenna points in the  $-x_s$  spacecraft direction. In this home position, the antenna dish frame is rotated about the body  $z$ -axis by 180 deg. from the spacecraft frame and the antenna central axis  $+x_D$  is in the  $-x_s$  direction.



**Figure 1. SDO Nominal Gimbal Orientation**

If no misalignments exist, rotation from the spacecraft frame to the dish frame is given by:

$$R_S^D = Y(\beta)Z(\alpha + \pi) \quad (1)$$

where  $Y$  and  $Z$  represent single axis rotations.

The azimuth gimbal may be misaligned with respect to the spacecraft, the elevation gimbal misaligned with respect to the antenna, and the

two gimbals misaligned with respect to each other. Including these misalignments, the rotation from the spacecraft frame to the dish frame becomes:

$$R_S^D = M_{el}^D Y(\beta) M_{az}^{el} Z(\alpha + \pi) M_S^{az} \quad (2)$$

where the rotations,  $M$ , represent small angle misalignments.

The gimbal angles are generally read from encoders that have potential bias and scale factor errors:

$$\begin{aligned} \alpha &= s_\alpha \alpha_0 + b_\alpha \\ \beta &= s_\beta \beta_0 + b_\beta \end{aligned} \quad (3)$$

where  $s$  represents scale factors,  $b$  represents biases and the variables with subscript “0” represent the measured values.

In the most general case there are 13 parameters that could be needed to completely specify the gimballed platform pointing direction given measured azimuth and elevation angles. These consist of the scale factors and biases for each gimbal and three parameters in each of the three misalignment matrices. Although the misalignments are small angles, they are rotated through large gimbal angles so the matrix multiplication order is generally important.

Each of the misalignments can be represented as products of three single-axis rotations in arbitrary order. Expanding the misalignments in this way gives:

$$R_S^D = X_{el}^D Z_{el}^D Y_{el}^D Y(s_\beta \beta_0 + b_\beta) Y_{az}^{el} X_{az}^{el} Z_{az}^{el} \dots Z(s_\alpha \alpha_0 + b_\alpha + \pi) Z_S^{az} X_S^{az} Y_S^{az} \quad (4)$$

In Eq. (4) the order in which the misalignments are decomposed is selected to place single-axis misalignment rotations adjacent to corresponding gimbal rotations. These rotations, about the same axes, can then be combined. The rotation angles in  $Y_{el}^D$  and  $Y_{az}^{el}$  can be combined with  $b_\beta$  to give an effective elevation bias designated  $\Delta\beta$ . An effective azimuth bias can be defined similarly. Equation (4) then simplifies to:

$$R_S^D = X_{el}^D Z_{el}^D Y(s_\beta \beta_0 + \Delta\beta) X_{az}^{el} \dots Z(s_\alpha \alpha_0 + \Delta\alpha + \pi) X_S^{az} Y_S^{az} \quad (5)$$

Equation (5) has only 9 independent parameters. Using lower case letters to represent the rotation angles and the subscript “ $el$ ” for the rotations from the elevation gimbal to the dish



frame, “*inter*” for the rotation from the azimuth to elevation gimbal and “*az*” for the rotations from the spacecraft frame to azimuth gimbal, the 9 parameters are:  $x_{el}$ ,  $z_{el}$ ,  $s_{\beta}$ ,  $\Delta\beta$ ,  $x_{inter}$ ,  $s_{\alpha}$ ,  $\Delta\alpha$ ,  $x_{az}$  and  $y_{az}$ . This reduction in the size of the parameter set needed for gimbal calibration follows the parameterization of the Upper Atmosphere Research Satellite (UARS) Solar-Stellar Pointing Platform (SSPP)[2].

The SDO HGAs have digital gimbal encoders so the scale factors can not vary and need not be determined on-orbit. The rotation about the dish axis,  $X_{el}^D$ , is unobservable for a symmetric dish and does not affect the antenna pointing direction. Thus, the required calibration reduces to solving for only 6 parameters.

### 3. Loss Function

In any calibration it is necessary to optimize some function of a measurable quantity with respect to the calibration parameters. The only measurable quantity that is available for HGA gimbal calibration is the power detected by a ground receiver. If the HGA is aimed at the receiver but there are errors in the gimbal parameters, the HGA will not point directly at the receiver and the detected power will be lower than expected. Differences between expected and observed power can be minimized with respect to the calibration parameters to determine their optimum values.

The HGA power is a function of the angle between the center of the antenna and the direction of the detector. The gain in decibels (dB) is given by:

$$G = 10 \log_{10} \left( \frac{P}{P_0} \right) \quad (6)$$

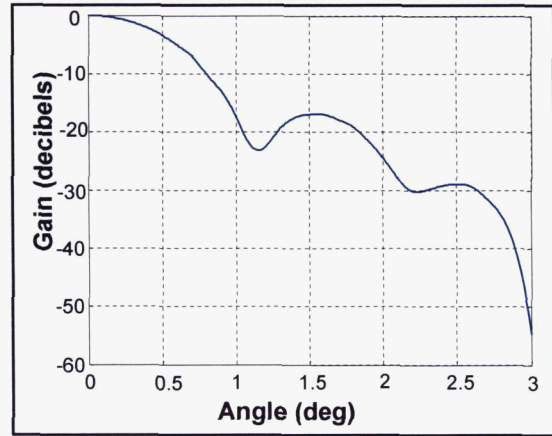
where  $P$  is the measured power and  $P_0$  is the reference power. Power measurements provide information about angular error only. They provide no information about the direction of the offset.

The power observed at the ground station is attenuated by the distance between SDO and the ground antenna by an inverse square law. Measured gain, including distance attenuation is given by:

$$\begin{aligned} G_d &= 10 \log_{10} \left( \frac{P}{P_0} \frac{r_0^2}{r^2} \right) \\ &= 10 \log_{10} \left( \frac{P}{P_0} \right) + 20 \log_{10} \left( \frac{r_0}{r} \right) \quad (7) \\ &= G + 20 \log_{10} \left( \frac{r_0}{r} \right) \end{aligned}$$

where  $r$  is the distance from SDO to the ground antenna,  $r_0$  is the distance corresponding to the reference power ( $P_0$ ),  $G$  is the gain assuming no distance attenuation, and  $G_d$  is the gain with distance attenuation.

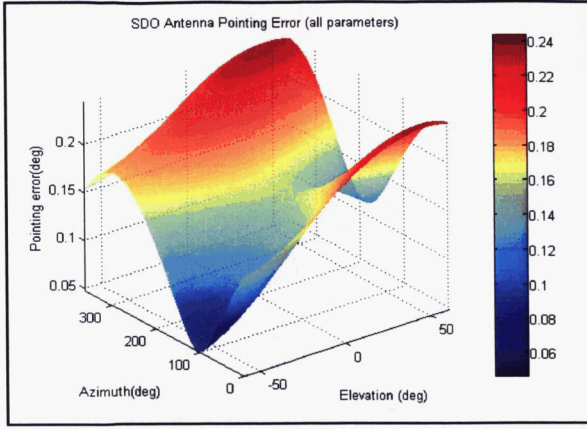
A typical gain function for an HGA such as those on SDO is shown in Figure 2.



**Figure 2. Gain Curve for HGA**

The simulated gain, as a function of angle, is interpolated from the measured gain curve of Figure 2 and corrected for distance attenuation using Eq. (7). The angle used is an error-angle which is the angle between the ideal vector from the spacecraft to the ground target (obtained using the spacecraft ephemeris) and the actual antenna  $x_D$ -vector. Conversion of these two vectors into a common frame must use a form of Eq. (5). The error-angle, and therefore the expected gain, is thus a function of the calibration parameters.

The angle between ideal and actual pointing directions due to misalignments is not a simple function of the gimbal angles and calibration parameters. One example of this angle, as a function of gimbal angles, is shown in Figure 3.



**Figure 3. SDO Antenna Pointing Angle with Uncompensated Azimuth and Elevation Gimbal Biases of 0.1 degrees and All Other Misalignments of 0.05 degrees**

To determine the calibration parameters, measurements of power are made with the spacecraft at different positions in its orbit (providing different azimuth and elevation angles) and with known, varied targeted offsets from the ground antenna. A loss function is constructed as the sum of squares of the differences between expected gains,  $G_{ex}$ , and observed gains  $G_{obs}$ , at the times  $t_i$ :

$$\mathcal{L} = \sum_i [G_{ex}(t_i) - G_{obs}(t_i)]^2 \quad (8)$$

The loss function (Eq. 8) is minimized with respect to each of the calibration parameters.

To compute  $G_{ex}$  as a function of the misalignments, the following procedure is used:

1. At each time, compute: the spacecraft position, the distance from spacecraft to the ground antenna, and the geocentric inertial (GCI) spacecraft-to-ground-antenna vector,  $\hat{A}_{GCI}$ , from the ephemeris,
2. Convert the GCI vector representing the SDO-to-ground antenna direction to the spacecraft body frame using the SDO attitude ( $M_{GCI}^S$ ) at that time,  $\hat{A}_S = M_{GCI}^S \hat{A}_{GCI}$ .
3. Compute the antenna pointing direction using the measured gimbal angles and the current parameter estimates using Eq. (5).
4. Compute the angle between the antenna pointing direction and the SDO-to-ground direction (from step 2).

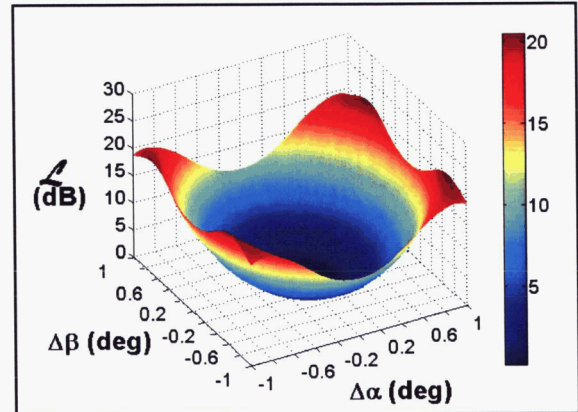
5. Interpolate the gain function (Figure 3) to this error angle and adjust for distance attenuation using Equation (7) to give  $G_{ex}$

The loss function is minimized with respect to the misalignment parameters using the MATLAB<sup>TM</sup> function *FMINSEARCH*. *FMINSEARCH* uses the SIMPLEX algorithm to find the minimum of a multivariate function.

#### 4. Simulation and Results

A simple simulator was developed that provides simulated receiver gain for any set of calibration parameters and commanded gimbal angles. Commanded gimbal angles were initially chosen to direct the antenna at the ground station with nominal alignments.

Because gain provides no information on the direction of an angular offset, the loss function tended to have indistinct minima. An example of a shallow, broad minimum is shown in Figure 4. The simulated calibration parameters used to generate this figure were nominal, except for 0.1 deg azimuth and elevation gimbal biases. The loss function is shown as a function of the estimated gimbal biases.



**Figure 4. Loss Function for Case with 0.1 deg Azimuth and Elevation Biases**

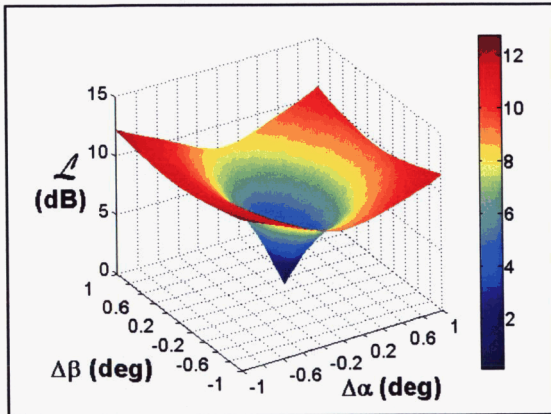
Even with non-zero values on all of the calibration parameters, all parameters can be determined. Minimization, however, requires very large amounts of data (samples at 10 second intervals for 2 days were used). The need for such large amounts of data can be reduced by varying the antenna pointing direction.

The loss function minimum can be made more distinct by intentionally moving the antenna in



a pattern of offsets relative to the ground antenna direction. A convenient pattern is a raster scan pattern where one gimbal angle is kept at a constant offset while the other is moved in steps. The first is then moved to a different position and the second moved in steps through its range. This pattern is repeated until the desired range of gimbal offsets for both gimbals has been exercised.

Using the same calibration parameters as used to generate Fig. 4, but applying a raster scan, the loss function becomes that shown in Figure 5. The raster scan had an amplitude of 2 deg on each axis, a step size of 0.1 deg, and was repeated throughout the simulation. Clearly, the loss function in the presence of a raster scan is easier to minimize than that in the nominal case.



**Figure 5. Loss Function for Case with 0.1 deg Azimuth and Elevation Biases with Intentional Gimbal Angle Offsets**

A set of simulations and tests has been performed to evaluate the calibration algorithm and develop a calibration plan. These are based on a nominal calibration scenario with the following conditions:

- Two orbits duration (48 hours)
- Power samples every 10 seconds
- Raster scan throughout the period
- Raster scan had a maximum offset of  $\pm 1$  degrees in each direction with 25 evenly spaced rows and columns
- At each position in the raster scan 10 seconds of data were observed. The complete raster scan has 625 samples and takes 6250 seconds to complete

- The raster scan pattern is repeated until the end of the calibration period (about 27 full raster scan cycles).
- “Truth” misalignments of 36 arcsec were applied to the six misalignment parameters that were corrupted.

Random noise (normal distribution) was applied to the measured power. The 1-sigma noise was 0.01% of the maximum power. This corresponds to about 1dB when the signal is -40dB.

The results showed excellent calibration capabilities. The errors between the simulated calibration parameters and their determined values are shown in Table 1. Table 1 also lists results from calibration cases in which the nominal scenario was modified as described in the first column of the table.

**Table 1. Parameter Magnitude Errors (arcsec) for Several Calibration Cases**

Case	$z_{el}$	$\Delta\beta$	$x_{inter}$	$\Delta\alpha$	$x_{az}$	$y_{az}$
Nominal	0.06	0.03	0.02	0.10	0.01	0.05
10 cycles of raster scan starting at orbit start	0.25	0.03	0.01	0.14	0.02	0.01
10 cycles of raster scan starting at 1/4 orbit	0.04	0.10	0.15	0.13	0.05	0.11
10 cycles of raster scan starting at 1/2 orbit	0.17	0.03	0.13	0.07	0.01	.08
One orbit duration	0.20	0.00	0.08	0.17	.00	0.06
One orbit duration: 5 rows and columns in raster scan	1.29	0.07	0.43	1.23	0.04	0.11
One orbit duration $\pm 0.5$ degrees raster scan	0.04	0.00	0.01	0.04	0.00	0.00
One orbit duration: $\pm 0.1$ degrees raster scan	0.04	.01	0.03	0.06	0.01	0.00
One orbit duration 10X nominal noise	0.29	0.44	0.07	0.47	0.03	0.00

A significant source of error in the SDO HGA calibration is expected to be thermal deformation of the antenna mast. The spacecraft maintains an attitude with the body  $x_s$  direction always pointing towards the Sun. The +x portion of the antenna mast between the body and the azimuth gimbal is always exposed to the Sun and may undergo thermal deformation. This deformation is not expected to be time-dependent so it acts as a constant misalignment in the calibration process. The portions of the mast between the azimuth gimbal and the an-



tenna itself are exposed to the Sun and are deformed in different directions depending on the gimbal angles. Variable exposure to the Sun is expected to produce time-dependent misalignments, and the static portion of the misalignments must be determined in the presence of these additional perturbations.

The portions of the gimbal mast expected to undergo time-dependent misalignment are the intergimbal strut and the elevation gimbal-to-antenna strut. The simulator modeled these misalignments as rotations perpendicular to the cross product of the strut axis and the Sun direction. The magnitude is proportional to the sine of the angle between the strut and the Sun direction. The total thermal deformation was simulated as the total angle budgeted by the spacecraft mechanical engineers for thermal deformation (0.02 deg) on the antenna mast.

Although the thermal bending varies with time, it approximately repeats from orbit to orbit. The deformation changes gradually with season.

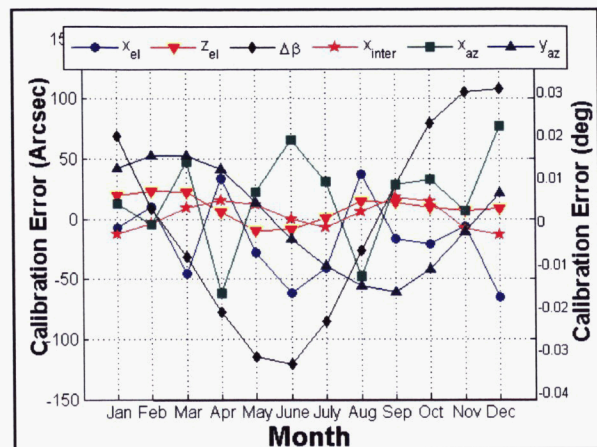
A worst case scenario was simulated and the HGA calibrated. In addition to the parameters of the nominal case, the worst case included thermal deformation and a 1 deg deployment error (in the  $x_{az}$  parameter). The simulation and calibration were repeated for conditions in each month of a year.

Errors in several of the calibration parameter estimates are found to be seasonally dependent as shown in Figure 6. In addition, some of the parameters (especially the elevation gimbal bias) are estimated with larger errors than their simulated values. This apparent amplification arises from large errors in other parameters influencing their estimated values. In any single-season data set, the thermal deformations cause the parameters to be not completely independent.

### Conclusions

For any gimballed platform, reduction of a general 13 parameter calibration space to 9 parameters can be performed without any loss of generalization. This reduction is achieved by formulating the pointing of the platform as a series of rotations taken in such an order that consecu-

tive rotations about common axes can be combined.



**Figure 6. Variation in Calibration Error with Season Using 1 deg. Deployment Error and a Thermal Distortion Model**

For the SDO HGAs it has been found that under all simulated conditions the calibration can be performed to achieve the required antenna pointing accuracy. This is true even if time-dependent thermal deformations occur.

Because of thermal distortions, calibration errors depend on time of year. It is possible that by using data from more than one season, even more accurate calibration results can be attained.

### Acknowledgements

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